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NUMBER THEORY.

217. (May, 1914.) Proposed by E. T. BELL, University of Washington.

(i) If r is a prime greater than 2, and $p \equiv 2^ar + 1$ is prime, the only solution, when n is greater than 2, of $x^n - y^n = p$, is $n = 3$, $x = 2$, $y = 1$.

(ii) The only primes that are simultaneously of the forms $4k + 1$ and $3^m - 2^m$ are 1 and 5.

(iii) Generalize (ii).

SOLUTION BY FRANK IRWIN, University of California.

(i) Since $x^n - y^n$ is divisible by $x - y$, we must, if it is to be a prime, have $x - y = 1$, or $x = y + 1$.

Again, n must be a prime; for $x^{st} - y^{st}$ is divisible by $x^s - y^s$, and cannot, therefore, be a prime.

Now $2^{ar} = p - 1 = x^n - y^n - 1 = (y + 1)^n - y^n - 1 = ny^{n-1} + \binom{n}{2} y^{n-2} + \dots + ny$; so that 2^{ar} is divisible by ny , since n , being a prime, divides each of the binomial coefficients. Consequently we must have $n = r$ (n is greater than 2 by hypothesis) and y a power of 2.

But by writing 2^{ar} as $x^n - (x - 1)^n - 1$, we may show, just as above, that x also is a power of 2.

Now as $x = y + 1$, these results are consistent with each other only if $y = 1$, $x = 2$.

In this last case we have $2^{ar} + 1 = 2^n - 1^n = 2^r - 1$, since $n = r$. Hence, $2^a(2^{r-a} - 1) = 2$, and, therefore, $a = 1$ and $2^{r-a} - r = 1$, that is, $2^{r-1} = r + 1$; so that $r = n = 3$.

(ii) $3^m - 2^m = (2 + 1)^m - 2^m = m2^{m-1} + \binom{m}{2} 2^{m-2} + \dots + m2 + 1$, and can be of the form $4k + 1$ only if m is even (unless $m = 1$, when $3^m - 2^m = 1$).

But if m is even, $= 2n$, say, $3^m - 2^m = 9^n - 4^n$ and is therefore divisible by $9 - 4$ or 5, the other factor being greater than 1, unless $n = 1$, and is, therefore, prime in this last case only.

(iii) A parallel argument leads to the result that the only primes that are simultaneously of the forms $a^2k + 1$ and $(a + 1)^m - a^m$ ($a > 1$) are 1 and, possibly, $(a + 1)^a - a^a$. This latter number may or may not be prime as the cases $a = 3, 4$ show.

Also solved by MARY E. CARTER.

221. (September, 1914.) Proposed by T. E. MASON, Bloomington, Indiana.

Find a number x such that the sum of the divisors of x is a perfect square. [CARMICHAEL, *Theory of Numbers*, p. 17.]

SOLUTION BY E. B. ESCOTT, Kansas City, Mo.

(1) Let x be prime. Then $x + 1 = r^2$, or $x = (r + 1)(r - 1)$. This is possible only if $r = 2$; otherwise, x is not prime. Hence, $x = 3$ satisfies the condition.

(2) Let $x = abc\dots$, where $a, b, c, \dots$ are different primes. Denoting the sum of the divisors of x by Σx ,

$$\Sigma x = (a + 1)(b + 1)(c + 1)\dots$$

Forming a table of factors of $a + 1$, it is very easy to pick out products of two or more which shall be square, e. g.,

x	Σx	x	Σx
2.11	6^2	7.97	28^2
2.47	12^2	.	.
11.47	24^2	.	.
5.23	12^2	.	.
5.53	18^2	2. 5.7	12^2
23.53	36^2	5. 7.11	24^2
7.17	12^2	5. 7.47	48^2
7.31	16^2	2.13.41	42^2
7.71	24^2	5.67.101	204^2
		.	.
		.	.

(3) Corresponding to a factor a^2 in the number, we have in the sum of the divisors $1 + a + a^2$, etc. By forming a table of $1 + a + a^2$, $1 + a + a^2 + a^3$, ... we can find examples where the number contains square factors, cube factors, ...

EXAMPLES:

x	Σx	x	Σx
$7^2.11^2.5.13$	798^2	$3^3.7.19$	80^2
$41^3.83$	2436^2	$2^2.5.41$	42^2
$41^3.5.13$	2436^2	.	.
$3^3.89$	60^2	.	.
		.	.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kans.

REPLIES.

31. What are the actual courses now offered in colleges and universities in this country for the preparation of teachers (1) for secondary schools, (2) for colleges? The discussion may well lead to the consideration also of what courses *should be offered* for the preparation of teachers of mathematics (1) for secondary schools, and (2) for colleges.

REPLY BY U. G. MITCHELL, University of Kansas.

In the American Report, Committee No. V, of the International Commission on the Teaching of Mathematics, published in 1911, we find (pp. 5-6) these statements:¹

"Twenty years ago no professional training of university grade existed in this country to prepare teachers of mathematics for secondary schools. At that time the young teacher's sole preparation for his work was the taking of as many academic courses as possible, plus, in some instances, a course on the history of education or some lectures on pedagogy. . . . About 15 years ago we find conditions throughout the country beginning to change in this respect. At least five different educational institutions had by this time (1895) established courses on the teaching of algebra and geometry, which, together with a course on general pedagogy, formed a certain professional training for high school teaching in mathematics. Up to 1900 only four other colleges are known to this committee to have added courses in the pedagogy of secondary mathematics to their programs.

"The past 10 years have shown far greater interest in pedagogical matters and a much more rapid growth in courses of this kind. At present (1910) about 25 other colleges in addition to those above mentioned, have developed such courses."

The writer has just completed an examination of the most recent catalogs from a selected list of 100 colleges and universities to see how many of them are now offering courses especially designed for students preparing to teach mathematics. The result shows that the change referred to above is continuing. Fifty-three of the 100 institutions are offering one or more courses of the following four different classes:

(1) *Courses especially designed for teachers of secondary mathematics.* Such courses were listed by 40 of the 100 institutions and were variously designated as "Teachers' Course," "Mathematical Methods," "History and Teaching of Mathematics," "Teachers' Course in Algebra and Geometry," etc. In five cases "reviews of secondary mathematics with special attention to methods"

¹ *Bulletin, 1911, No. 12, U. S. Bureau of Education* (Washington, D. C.).